This article discusses how an active learning environment was created in a community college class and a high school mathematics class using the same approach. In both settings the NCTM and AMATYC Standards, such as problem solving, communication, use of technology and alternate assessment, were fully embraced. The goal was to have students take charge of their learning, while the instructors took a secondary role to the students. The lecture time was reduced to a fraction of the class time, while collaborative learning time was maximized. The author's experience indicates that collaborative learning alone is not the solution, but integrating all of NCTM/AMATYC suggestions at every class is. Interviews with students, at different stages of the course, were used to demonstrate how this pedagogy empowered students.

Students not majoring in mathematics and sciences often perceive mathematics as an abstract, rigid field that has little or no application in daily life. Mathematics teachers, on the other hand, are increasingly dissatisfied by student achievement and lack of conceptual understanding. Thus, current mathematics education leaves both parties discontent. Still, in many secondary schools and college mathematics courses, lecturing remains the most common form of instruction (Wilson, 1996). In this form of teaching, the students are mostly passive (Wilson, 1994; Roth, 1994). This pedagogy assumes that a transfer of knowledge from an expert to a passive learner is possible; however, research indicates that this kind of instruction causes students to learn facts in isolation, often forgetting the reasoning behind the explanations (Roth, 1994). Research indicates that this approach neither encourages nor enhances learning that fosters conceptual understanding (Wilson, 1994; Thorton & Sokoloff, 1989; Hestenes, Wells, & Swackhamer, 1992). Brownell (1987) points out that majority of mathematics instruction is based on giving an algorithm and later asking student to practice it, making mathematics education similar to following a recipe, where the reasoning behind a procedure is often lost. As instructors, we want students to make connections between mathematical ideas, but we do not always provide them with opportunities to do so.

With the publications of NCTM Standards (1989) and AMATYC Standards (1995) there has been an increased emphasis on students discovering the mathematics they are learning, making hypotheses and conjectures and then testing them. Countryman (1992) states that students can construct mathematics "only by exploring, justifying, representing, discussing, using, describing, investigating, predicting, in short by being active in the world" (p. 2). The need to teach mathematics through real-life applications, strengthening problem-solving skills, using collaborative learning as a tool, increasing communication skills in the area of
mathematics and alternative assessment strategies have also been stressed frequently in recent literature (NCTM Standards, AMATYC Standards, 1995).

Despite these recommendations, the predominant form of education in many high schools and community colleges remains lectures. Although, collaborative work is often incorporated, what is typically discussed in collaborative groups is a reiteration of the steps to a procedure already outlined by the instructor. Students work on examples identical to those the instructor has previously solved, simply using different numbers. Even if the NCTM and AMATYC recommendations are followed, not all the recommendations are followed in the same course all of the time.

**Approach**

This paper discusses how the NCTM and AMATYC suggestions were utilized in an urban community college and a high school to teach a prealgebra course that included arithmetic, basic geometry and introductory algebra topics. In both settings, students had seen the topics repeatedly, but had not sufficiently mastered them.

The first assertion of our teaching philosophy was that students would learn what they perceived as valuable and get motivated if they saw the relevance of the concepts in daily life; therefore, the approach used must first make a connection between the mathematics to be learned and its application to a real situation. Most textbooks and classroom instruction start with an abstract treatment of the topic followed by drills; only at the end do they discuss its applications. We switched the order, first giving an example of a situation, whose solution required the mathematics we wanted students to learn. We used a textbook grounded in this philosophy. Each activity depicted a real-life scenario and data (on such problems, since real-life data can be cumbersome to manipulate, students were allowed to use calculators). Nearly all the problems discussed were based on consumer mathematics, finance, environment, and so on. The point was that mathematics naturally occurs in many areas of life. Thus the value of why the concept had to be learned became apparent to the student. For instance, one class activity asked students to analyze the mathematical information on an electric bill, leading them to discover how rates, percentages, graphs, and estimation skills were used. While some problems had unique answers, some were open ended and required students to make informed assumptions.

Students often dread word problems. Part of the reason is that problem solving requires genuine thinking, and, though some problems can be solved through an algorithmic approach, if problems are varied enough, it requires more diverse analytical thinking skills. Since each activity used in class was formulated as a real life problem, students became exposed to word problems on a daily bases. Each problem, they encountered, was a word problem, in which they needed to identify relevant and not-so-relevant information. At first, students tended to regard every piece of information in the problem of equal importance (Larkin et. al, 1980; Norris, 1985). However, as students' prob-
Problem solving skills developed, their perception of what was meaningful also changed. In our pedagogy, rather than placing the emphasis on the solution to a problem, we encouraged students to think about the approach they used. As the course progressed, so did students' problem solving skills. Moreover, multiple representations - verbal, numerical, algebraic and graphical - were encouraged throughout the course, making it easier for students to make connections.

The second assertion of our teaching philosophy was that in order to create an active learning environment, the instructor should take a secondary role to the students. This was only possible under the belief that students are capable of more than they fathom. As instructors, unless we set our expectations high, students have no chance to rise to the occasion. The instructor viewed himself/herself more as a coach rather than as a dispenser of knowledge. In order for students to investigate, discuss, develop and evaluate problem-solving approaches, cooperative groups were formed. Groups comprised three or four students and were heterogeneous in ability, often bringing together a high-ability, a medium-ability and a low-ability student. This was inline with recommendations on effective cooperative groups (Johnson, et. al., 1984; Heller and Hollabaugh, 1992). Being in groups forces students to work jointly on problems, pinpoint the problem's underlying assumptions, clarify the material for one another, and give each other feedback (Slavin, 1996). Students at the basic level tend to stick to one familiar approach and not attempt to understand different methods, claiming it is confusing, but this setting required them to analyze alternate approaches and consider their peers' strategies.

To create an active learning environment, the lectures were limited to a fraction of the class time. Unlike many classes where the instructor solves a problem and asks students to work on similar problems, in this class, students were initially asked to work on a new problem with their group members. Students were encouraged to be independent learners and first try to come up with an answer in their own group. The instructor was consulted only as a last resort. Even then, the instructor helped clarify students' thinking through guided questions rather than a direct answer. Since groups worked at their own pace, occasionally one group would be far ahead in the material while another fell behind. Then the members of the former group would work with the members of the later as consultants. They would not solve the problem for their classmates but gave hints and helped clarify their thinking process. This practice gave advanced students something meaningful to do, while acknowledging their work. The instructors, when needed, functioned in this role as well.

Cooperative groups also served other purposes besides providing an active learning environment. The first purpose was to create a support group for students. Secondly, it created peer pressure. Each group was given the right to fire any of its members who were not carrying their weight. Thus, there could be consequences for not coming to class on time or not doing or at least making a serious attempt to do home-
work. When a member was fired (as in the case of one group) the instructor did not get involved. Just like any person who gets fired from a job in the real world, the student needed to convince another group to take him/her on as a member. Instead of a singular grade-based pursuit, motivation came from multiple real-life pressures within peer groups.

Another advantage of group work was that it allowed students to communicate their mathematical ideas, as suggested by NCTM and AMATYC Standards. Communication of mathematical ideas was not limited to conversations within the groups. The textbook - more like a work-book - encouraged students to formulate mathematics in a way that was understandable to them, giving them a chance to organize and internalize their thoughts. One college student puzzled, “there was a rule that had to do with absolute values that one of my teachers told me. What was it?” The instructor suggested the student write it as it made sense to him; since if he was told the “rule,” most probably this instructor would not fare any better than the previous.

**Reflective Writing**

Writing assignments also encouraged conceptual understanding. Rothstein & Rothstein (2007) state that the “... benefits of combining mathematics with writing include promoting the ability to analyze, compare facts, and synthesize information” (p.22).

These assignments asked students to explain their reasoning and challenged them with “why” questions that encouraged them to reflect on the reasons behind mathematical algorithms: “Why do we need to find common denominators when adding/subtracting fractions but not when multiplying/dividing?” Such probing writing urged students to consider whether the answer found was reasonable; it developed conceptual understanding and encouraged connections and generalizations that the students might not otherwise have made. Thus, students were better able to organize their ideas and reflect on what they learned (Burns, 2004). While group discussions encouraged multiple problem-solving strategies, writing exercises deepened students’ abilities to articulate mathematical ideas and critical thinking skills. Since reasoning skills were at the core of this course, student performance was far better than those where students perceive mathematics as a collection of rote, disconnected and decontextualized algorithms. In this course, students learned when and why to apply different approaches.

**Alternative Assessment**

In the community college, assessment consisted of individual and group exams, multiple workbook evaluations, a project and classroom participation. Students ultimately presented their best work in the form of a portfolio for which the instructor decided on the minimum number of exams to be included. In the high school, the same categories applied (instead of a portfolio, all work was taken into account). Immediately after each group exam, a random member of each group was asked to explain a particular solution on the board. His/her individual grade on this presentation was averaged with the group grade on
the exam. Each in-class exam consisted of two parts: first, drill questions without calculators, and second, word problems using real-life data with calculators. This ensured students' ability to do both basic operations and to solve problems. Through multiple types of assessment, a student's knowledge and capabilities were better measured. The projects gave students the chance to demonstrate a multitude of mathematical proficiencies, as well as contextual writing and presentation skills.

Resistance to Change

The community college students came to the course having failed mathematics many times. This led to frustration, anxiety and poor self image. They were accustomed to having the instructor solve a problem and being given a similar problem to solve. If previous teaching style of lecturing interspersed with cooperative learning had worked, students would not have needed remediation on arithmetic skills at the community college level. Students resisted being asked to solve new types of problems. For many, being given a problem whose solution they had not watched before was unacceptable. They demanded that the instructor show them a similar problem, without even attempting to solve it. Change, even if for the better, was difficult.

In the high school, though not as much as in the community college class, there was some level of discomfort as well. Some students found explaining their mathematical ideas in writing difficult and boring. They did not see a need for it, and thought obtaining a solution was the final step. They often said “I did it but I can’t explain it” and asked the teacher to do the explaining. Still, the school administration’s support, seeing fellow students in other sections work within use a similar pedagogy and students’ less-rigid definition of what entailed teaching helped this group make a smoother transition. While community college students complained that they should not be expected to solve a new problem, it was easier to encourage high school students to embrace it as a challenge. One explanation of this difference in behavior was that the average age in the high school class was a few years younger than that of the community college, where there were older students returning to school after many years and more steadfast in their expectations of how they should be taught. Furthermore, there was more time for high-school students to adapt to the new pedagogy. While community college students had thirteen weeks with their instructor, high school students had an entire year. Also, the former group felt more pressure to produce passing grades and exhibited a more pervasive math anxiety, and this likely contributed further to the difference in behavior.

Students’ Thoughts

At the beginning, most community college students hated this approach. Their plea was “Just show us how to get the answer.” To many, this approach seemed like a waste of time. The students were video taped at regular intervals and their perceptions were recorded. The following comments were from these videos taken during the first week of the semester.

Monica: I am 38 years old. I have been
out of school for ages. I am afraid I will fail, because I am no good in math. Never been, never will. I have a poor memory.

**Instructor:** You can't start by assuming you will fail. You need to keep an open mind. As for your poor memory, I have a terrible memory myself.

**Monica:** But you are a math professor!

**Instructor:** Exactly. You do not need a good memory to be good in math. You do not memorize mathematics. You understand it; you reason through it.

**Monica:** I always tried to memorize math. There are formulas.

**Instructor:** Formulas are a very little part of mathematics. Most formulas you will use in this course, like perimeter formulas, should make sense too.

Another conversation is given below.

**Carmela:** I don't know why you expect me to be able to solve problems I've never seen before.

**Instructor:** Because I believe you can! Though you have not seen this particular problem, you have seen all the concepts needed in its solution.

**Carmela:** It is difficult. Math is difficult for me. I wish you did a problem like it, then I will be able to follow the steps and solve others. That's what my previous teachers did. That's how I learn best.

**Instructor:** In the past, did you have the experience that you understood things in class, but could not solve problems when you went home?

**Carmela:** Yes.

**Instructor:** That is because you were following your teacher's thinking and were not genuinely thinking yourself. You were sort of mimicking your instructor's steps. If you really understood each step, you would not be confused when you went home. My aim is that this time you will not mimic any one and you will discover how to do it. When you discover something, it makes sense. You do not get confused later on and you do not forget.

Here is how one student perceived group work.

**Md:** I don't like groups. If I do not know the solution and my group does not, then how can we help each other?

**Instructor:** You all know more than you think. Maybe talking to each other, you will find a solution. Even if not all of you have all the pieces, you may each have some part of the solution. Talking together will help you bring your ideas out. Please be patient, and give this method a chance. At the end of the semester, students were interviewed by mathematics department faculty and these interviews were also videotaped.

**Monica:** I really learned math. I was so afraid first, because I am older and been away from school. As I started doing the problems, my confidence came back. At first my group was helping me a lot and I felt bad because I needed so much help. But then when we were doing percents, I was helping all of them. So it is give and take. We were there
for each other. I learned math and made friends. Not only in class, but we would get together outside of class too. Maxim and I both had a free period at the same time and we met. Gabriel would come over to my house on weekends and we would study. There was so much support.

Angela: The course was great, though we complained a lot at first. Now I can say I understand math. Word problems do not scare me any more. Before, I was afraid. Now I tell my mom - she is bad with math (laughs) and my other friends “Come I will teach you math.” I love math, I am good at it. I never thought, I would say that!

Carmela: Before, I did not know why we studied math. I needed it to graduate. With the problems we worked on, its use is right in your face!

At the high school, students were not video-taped but had informal talks with the instructor, who wrote synopses of the dialogues and evaluated students’ progress. Some examples are given below:

Nancy, who was a weak student, showed a marked improvement this year. She is more willing to show her solutions on the board and explain her thought process. Her pride and ownership are clearly visible. She also indicated she enjoyed math class more this year.

Jose commented that now he knows why his teachers kept saying mathematics was important. He said that before the course he “really did not think this stuff [mathematics] was actually used.”

Madelynn said she was no longer afraid of word problems. She also noted that she had thought math problems had a unique correct answer. She was surprised that on some problems where they had to make some assumptions, multiple correct answers were possible.

Damián said he enjoyed coming up with his own rules for doing math. Though, at the beginning, he questioned why he had to write in a mathematics course and preferred to “just do it” rather than “explain” his solution, at the end he said it was “OK” and thought it helped his friends too. He admits that he found it a bore at the beginning to always talk about his thinking, but acknowledges that at times it helped him catch his mistakes. “It was cool!” is how he summarized the course.

Conclusion

We summarize this pedagogy as empowering; it allows students to take charge of their education to become active learners. At the base of this pedagogy lies the belief that you need to expect more from students and trust that they can rise to the standards you put.

Unlike most classrooms, where one aspect of reform is emphasized in a given course, our pedagogy is to integrate all aspects of reform into the course at all times. We believe that was the basis of the NCTM and AMATYC Standards in the first place and our experience indicates it makes a difference.
References


